Day 5, SPb SU LOUD Enough Contest 2

February 4, 2023

Courses

- We may write natural dynamic programming $dp[i][j]$ with states - money spend and IQ.

Courses

- We may write natural dynamic programming $dp[i][j]$ with states - money spend and IQ.
- Tranisitions - $dp[i][j] = \sum n[i][j] * dp[i-c][j-d]$. Time is $n^2(\max c)^2$.

Courses

- Let's look to the generating function $F_i(x) = \sum dp[i][j]x^j$

Courses

- Let's look to the generating function $F_i(x) = \sum dp[i][j]x^j$
- We need to calculate sum of some coefficients of F_i for every $i \le n$.

Courses

- Let's look to the generating function $F_i(x) = \sum dp[i][j]x^j$
- We need to calculate sum of some coefficients of F_i for every $i \leq n$.
- Let's remember the Fourier transform. This sum of coefficients may be rewritten as $\sum_t c_t F_i(\epsilon_t)$, where ϵ_t - roots of unity.

Courses

- Let's look to the generating function $F_i(x) = \sum dp[i][j]x^j$
- We need to calculate sum of some coefficients of F_i for every $i \leq n$.
- Let's remember the Fourier transform. This sum of coefficients may be rewritten as $\sum_t c_t F_i(\epsilon_t)$, where ϵ_t - roots of unity.
- But how the $F_i(\epsilon_t)$ looks like?

Courses

- Consider the generating function $F_i(\epsilon_t)y^i$.

Courses

- Consider the generating function $F_i(\epsilon_t)y^i$.
- It is some rational function with degree of denominator and numerator no more then $\max c$.

Courses

- Consider the generating function $F_i(\epsilon_t)y^i$.
- It is some rational function with degree of denominator and numerator no more then $\max c$.
- We just need to calculate linear combination of such rational functions, it can be done by divide-and-conquer. It is $O(n(\max c)\log^2 n)$.

Young

- We have graph, vertices are Young tableuxes, edge between them - if they differ in one cell. The task is to calculate number of ways to reach one vertex from another in the given number of steps.

Young

- We have graph, vertices are Young tableuxes, edge between them - if they differ in one cell. The task is to calculate number of ways to reach one vertex from another in the given number of steps.
- Lets say that the edge goes up, if it decreases number of cells, else down.

Young

- We have graph, vertices are Young tableuxes, edge between them - if they differ in one cell. The task is to calculate number of ways to reach one vertex from another in the given number of steps.
- Lets say that the edge goes up, if it decreases number of cells, else down.
- Main observation - for every vertex number of edges going up is less by 1 then number of edges going down.

Young

- We have graph, vertices are Young tableuxes, edge between them - if they differ in one cell. The task is to calculate number of ways to reach one vertex from another in the given number of steps.
- Lets say that the edge goes up, if it decreases number of cells, else down.
- Main observation - for every vertex number of edges going up is less by 1 then number of edges going down.
- Moreover, if 2 vertices have common neighbor, they have both common up neighbor and down neighbor.

Young

- Let's call a signature sequence of letters U and D. Path satisfies it, if in letter U it goes up, and in letter D it goes down.

Young

- Let's call a signature sequence of letters U and D. Path satisfies it, if in letter U it goes up, and in letter D it goes down.
- Let's calcualte number of paths of some signature ($F(S)$). Suppose it has two adjacent letters DU in this order (sequence is S_1DUS_2). Then we can see that $F(S) = F(S_1UDS_2) + F(S_1S_2)$.

Young

- Let's call a signature sequence of letters U and D. Path satisfies it, if in letter U it goes up, and in letter D it goes down.
- Let's calcualte number of paths of some signature ($F(S)$). Suppose it has two adjacent letters DU in this order (sequence is $S_1 DUS_2$). Then we can see that $F(S) = F(S_1 UDS_2) + F(S_1 S_2)$.
- In this manner we can express all $F(S)$ by linear combination of $F(UU \dots UDD \dots D)$. Coefficients in expression have combinatorial sence.

Young

- Let's call a signature sequence of letters U and D. Path satisfies it, if in letter U it goes up, and in letter D it goes down.
- Let's calcualte number of paths of some signature ($F(S)$). Suppose it has two adjacent letters DU in this order (sequence is $S_1 DUS_2$). Then we can see that $F(S) = F(S_1 UDS_2) + F(S_1 S_2)$.
- In this manner we can express all $F(S)$ by linear combination of $F(UU \dots UDD \dots D)$. Coefficients in expression have combinatorial sence.
- Then we can represent sum of all $F(S)$ by linear combination of $F(UU \dots UDD \dots D)$. Coefficients may be found explicitly.

Young

- Values of all $F(UU\dots UDD\dots D)$ can be found in dynamic programming, where states are Young subdiagrams of one of the two initial diagrams.

Young

- Values of all $F(UU \dots UDD \dots D)$ can be found in dynamic programming, where states are Young subdiagrams of one of the two initial diagrams.
- Asymptotics is $O(k + $"Number of Young subdiagrams of the given one"$)$, which is small enough.

Endless road

- We have 3 guys, every second random one moves 1 step in the positive direction. Task is to calculate expectation of maximal coordinate of them.

Endless road

- We have 3 guys, every second random one moves 1 step in the positive direction. Task is to calculate expectation of maximal coordinate of them.
- This number is indeed $F(n) = \sum_{x+y+z=n} max(x + a, y + b, z + c)\frac{n!}{3^n x!y!z!}$.

Endless road

- We have 3 guys, every second random one moves 1 step in the positive direction. Task is to calculate expectation of maximal coordinate of them.
- This number is indeed $F(n) = \sum_{x+y+z=n} max(x+a, y+b, z+c)\frac{n!}{3^n x!y!z!}$.
- Consider simpler sum: $S(n) = \sum_{x+y+z=n, x+a \geq y+b, x+a \geq z+c} \frac{1}{x!y!z!}$. Logic wih initial one is the same.

Endless road

- We have 3 guys, every second random one moves 1 step in the positive direction. Task is to calculate expectation of maximal coordinate of them.
- This number is indeed $F(n) = \sum_{x+y+z=n} max(x+a, y+b, z+c)\frac{n!}{3^n x!y!z!}$.
- Consider simpler sum: $S(n) = \sum_{x+y+z=n, x+a \geq y+b, x+a \geq z+c} \frac{1}{x!y!z!}$. Logic wih initial one is the same.
- Let's try to calculate $S(n)$ from $S(n-1)$.

Endless road

- Denote $A(x,y,z) = \frac{1}{x!y!z!}$. Then we can see that $nA(x,y,z) = A(x,y,z-1) + A(x,y-1,z) + A(x,y,z-1)$. That is, we can see that if we sum up this equality for whole region, in the right side will be almost $3S(n-1)!$

Endless road

- Denote $A(x, y, z) = \frac{1}{x!y!z!}$. Then we can see that $nA(x, y, z) = A(x, y, z-1) + A(x, y-1, z) + A(x, y, z-1)$. That is, we can see that if we sum up this equality for whole region, in the right side will be almost $3S(n-1)$!
- Problems will be only on borders. So we need to decrease or increase $S(n)$ by some sums where $x + a$ is almost equal $y + b$ or $z + c$.

Endless road

- Denote $A(x, y, z) = \frac{1}{x!y!z!}$. Then we can see that $nA(x, y, z) = A(x, y, z - 1) + A(x, y - 1, z) + A(x, y, z - 1)$. That is, we can see that if we sum up this equality for whole region, in the right side will be almost $3S(n - 1)$!
- Problems will be only on borders. So we need to decrease or increase $S(n)$ by some sums where $x + a$ is almost equal $y + b$ or $z + c$.
- It is the sums like $\sum_{x+a \geq y+b} B(x, y)$, where $B(x, y) = \frac{1}{x!(x+t)!y!}$

Endless road

- Denote $A(x, y, z) = \frac{1}{x!y!z!}$. Then we can see that $nA(x, y, z) = A(x, y, z-1) + A(x, y-1, z) + A(x, y, z-1)$. That is, we can see that if we sum up this equality for whole region, in the right side will be almost $3S(n-1)$!
- Problems will be only on borders. So we need to decrease or increase $S(n)$ by some sums where $x + a$ is almost equal $y + b$ or $z + c$.
- It is the sums like $\sum_{x+a \geq y+b} B(x, y)$, where $B(x, y) = \frac{1}{x!(x+t)!y!}$
- How to calculate this? Almost the same! We can express $B(x, y)$ as a linear combination of $B(x-1, y), B(x, y-1), B(x, y-2)$ and pass to the borders, which are where all $x + a, y + b, z + c$ are almost equal, and these are just binomial coefficients.

Endless road

- Denote $A(x,y,z) = \frac{1}{x!y!z!}$. Then we can see that $nA(x,y,z) = A(x,y,z-1) + A(x,y-1,z) + A(x,y,z-1)$. That is, we can see that if we sum up this equality for whole region, in the right side will be almost $3S(n-1)$!
- Problems will be only on borders. So we need to decrease or increase $S(n)$ by some sums where $x+a$ is almost equal $y+b$ or $z+c$.
- It is the sums like $\sum_{x+a \geq y+b} B(x,y)$, where $B(x,y) = \frac{1}{x!(x+t)!y!}$
- How to calculate this? Almost the same! We can express $B(x,y)$ as a linear combination of $B(x-1,y), B(x,y-1), B(x,y-2)$ and pass to the borders, which are where all $x+a, y+b, z+c$ are almost equal, and these are just binomial coefficients.
- Time is $O(n)$ with big constant. This idea works for any number of points.

Sets may be good

- Given a graph, we need to calculate number of sets of vertices with even number of edges between them.

Sets may be good

- Given a graph, we need to calculate number of sets of vertices with even number of edges between them.
- We can look at it as follows: we have some quadratic polynomial $P(x_1, \dots x_n)$ over $\mathbb{F}_2$, we need to calculate number of solutions of equation $F(x_1, \dots, x_n) = 0$.

Sets may be good

- Given a graph, we need to calculate number of sets of vertices with even number of edges between them.
- We can look at it as follows: we have some quadratic polynomial $P(x_1, \ldots x_n)$ over $\mathbb{F}_2$, we need to calculate number of solutions of equation $F(x_1, \ldots, x_n) = 0$.
- We can suppose F does not contain term x_i^2, cause $x_i^2 = x_i$

Sets may be good

- Given a graph, we need to calculate number of sets of vertices with even number of edges between them.
- We can look at it as follows: we have some quadratic polynomial $P(x_1, \ldots x_n)$ over $\mathbb{F}_2$, we need to calculate number of solutions of equation $F(x_1, \ldots, x_n) = 0$.
- We can suppose F does not contain term x_i^2, cause $x_i^2 = x_i$
- Let's take one variable x_1. If there is no monomials with it, then in doesn't have any influence to polynomial.
- In the other case, we can say that $P = x_1 L(x_2, \ldots x_n) + Q(x_1, \ldots, x_n)$, where L is linear.

Sets may be good

- Lets divide all variable substitutions on 2 cases - $L = 0$ and $L = 1$.

Sets may be good

- Lets divide all variable substitutions on 2 cases - $L = 0$ and $L = 1$.
- In the second variant, there is equal number of solutions with $P = 0$ and $P = 1$, cause when we change x_1, P is also changing.

Sets may be good

- Lets divide all variable substitutions on 2 cases - $L = 0$ and $L = 1$.
- In the second variant, there is equal number of solutions with $P = 0$ and $P = 1$, cause when we change x_1, P is also changing.
- So we reduce the problem to the following one: how many solutions of system $P = 0, L = 0$ are.

Sets may be good

- Lets divide all variable substitutions on 2 cases - $L = 0$ and $L = 1$.
- In the second variant, there is equal number of solutions with $P = 0$ and $P = 1$, cause when we change x_1, P is also changing.
- So we reduce the problem to the following one: how many solutions of system $P = 0, L = 0$ are.
- But we can make a variables changing to make L new variable. So we reduced the problem to the same problem with less number of variables. We can continue this process further.

Sets may be good

- Lets divide all variable substitutions on 2 cases - $L = 0$ and $L = 1$.
- In the second variant, there is equal number of solutions with $P = 0$ and $P = 1$, cause when we change x_1, P is also changing.
- So we reduce the problem to the following one: how many solutions of system $P = 0, L = 0$ are.
- But we can make a variables changing to make L new variable. So we reduced the problem to the same problem with less number of variables. We can continue this process further.
- Time is $O(n^3)$.