Statement

Problem A: Bijection

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- Combinations, $\mathrm{choose}(6, 3) = 20$:
  "RRRUUU", "RRURUU", "RRUURU", "RRUUUR", "RURRUU",
  "RURURU", "RURUUR", "RUURRU", "RUURUR", "RUUURR",
  "URRRUU", "URRURU", "URRUUR", "URURRU", "URURUR",
  "URUURR", "UURRUU", "UURURU", "UURUUR", "UUURRR".

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  "URRRUU", "URRURU", "URRUUR", "URURRU", "URURUR",
  "URUURR", "UURRUU", "UURURU", "UURUUR", "UUURRR".
- Regular bracket sequences, $C_3 = 5$:
  "((()))", "(()())", "(())()", "()(())", "()()()".
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Enumeration

Solution 1: just number the objects lexicographically.

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Requirements:

- big integers
- combination $\longleftrightarrow$ its number
- bracket sequence $\longleftrightarrow$ its number

Tedious but straightforward.
Note: using a non-default language might speed up development.

Proper Bijection

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From path to bracket sequence.

- Write down the path, add an "U" at the end.
- Walk along the path and maintain the balance, where "R" adds $+1$ and "U" adds -1.
- Mark the first time when the balance was the least possible. The number k is the marked position minus one. The character before the marked position is "U".
- Now loop the path.
- Starting from the marked position, take $2n$ characters. Transform "R" $\to$ "(" and "U" $\to$ ")". This is guaranteed to be a regular bracket sequence.

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- Walk along the path and maintain the balance, where "R" adds $+1$ and "U" adds -1: $\{+1, 0, -1, 0, -1, 0, -1\}$.
- Mark the first time when the balance was the least possible: "RUU|RURU". The number k is the marked position minus one: $k = 3 - 1 = 2$. The character before the marked position is "U".
- Now loop the path: "$\underbrace{\text{RUU|RURU}}\underbrace{\text{RUU|RURU}}$...".
- Starting from the marked position, take $2n$ characters. Transform "R" $\to$ "(" and "U" $\to$ ")". This is guaranteed to be a regular bracket sequence: "RURURU" $\to$ "()()()".

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From bracket sequence to path.

- Write down the sequence, add an “)” at the end.
- Now loop the sequence.
- Mark the position $2n - k$.
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From bracket sequence to path, example: "()()()", $k = 2$.

- Write down the sequence, add an ")" at the end: "()()())".
- Now loop the sequence: "$\underbrace{()()())}\underbrace{()()())}$...".
- Mark the position $2n - k = 6 - 2 = 4$: "$\underbrace{()()|())}\underbrace{()()())}$...".
- Starting from the marked position, take $2n$ characters. Transform "(" $\to$ "R" and ")" $\to$ "U": "())()(" $\to$ "RUURUR".

Other Bijections

Solution 3: other bijections are possible.

For example, there is one based on *exceedance*, see here:
`https://en.wikipedia.org/wiki/Catalan_number#Third_proof`.

Statement

Problem B: Rectangle Tree

In this problem, you have to rebuild a binary rooted tree of combinatorial rectangles so that it has logarithmic height compared to its size.

Definitions

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Just like a rectangle consisting of board squares is (consecutive subset) $\times$ (consecutive subset), a *combinatorial rectangle* is (arbitrary subset) $\times$ (arbitrary subset).

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In this problem, you have to rebuild a binary rooted tree of combinatorial rectangles so that it has logarithmic height compared to its size.

Just like a rectangle consisting of board squares is (consecutive subset) $\times$ (consecutive subset), a *combinatorial rectangle* is (arbitrary subset) $\times$ (arbitrary subset).

We have an $n \times n$ board consisting of colored squares.
We also have a rooted tree of combinatorial rectangles.
The root corresponds to the whole board.
Each vertex either is a leaf or has two children.
Children form a partition of the parent.
For each leaf, the cells it contains are colored the same.

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Our task is to construct a tree with the same properties.
Its size must not increase too much.
Its height must be logarithmic in terms of size.

Solution: Restricton

Let us define *restriction* of a tree to a combinatorial rectangle $A \times B$:

- Intersect all combinatorial rectangles in the tree with $A \times B$.
- Remove all empty rectangles.
- If only one child is not empty, promote it to parent.

Solution

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Now, construct the new balanced tree as follows:

- Find the center C of the tree: its subtrees are of size $\leqslant S/3$, but its own subtree is of size $\geqslant S/3$.
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- Then, the new root is $U \times U$.
- The left child is $A \times U$.
- Its children are $A \times B$ and $A \times (U \setminus B)$.
- The right child is $(U \setminus A) \times U$.

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- For each of the three lower vertices, restrict the whole initial tree to its combinatorial rectangle, then construct a balanced subtree from it, with respective rectangles instead of $U \times U$.

Further Reading

Further reading: the topic can be googled as "balancing communication protocol".

Statement

Problem C: Fastest Arrival

In this problem, you have to find an integer point in a disc that is closest to a given point on the plane.

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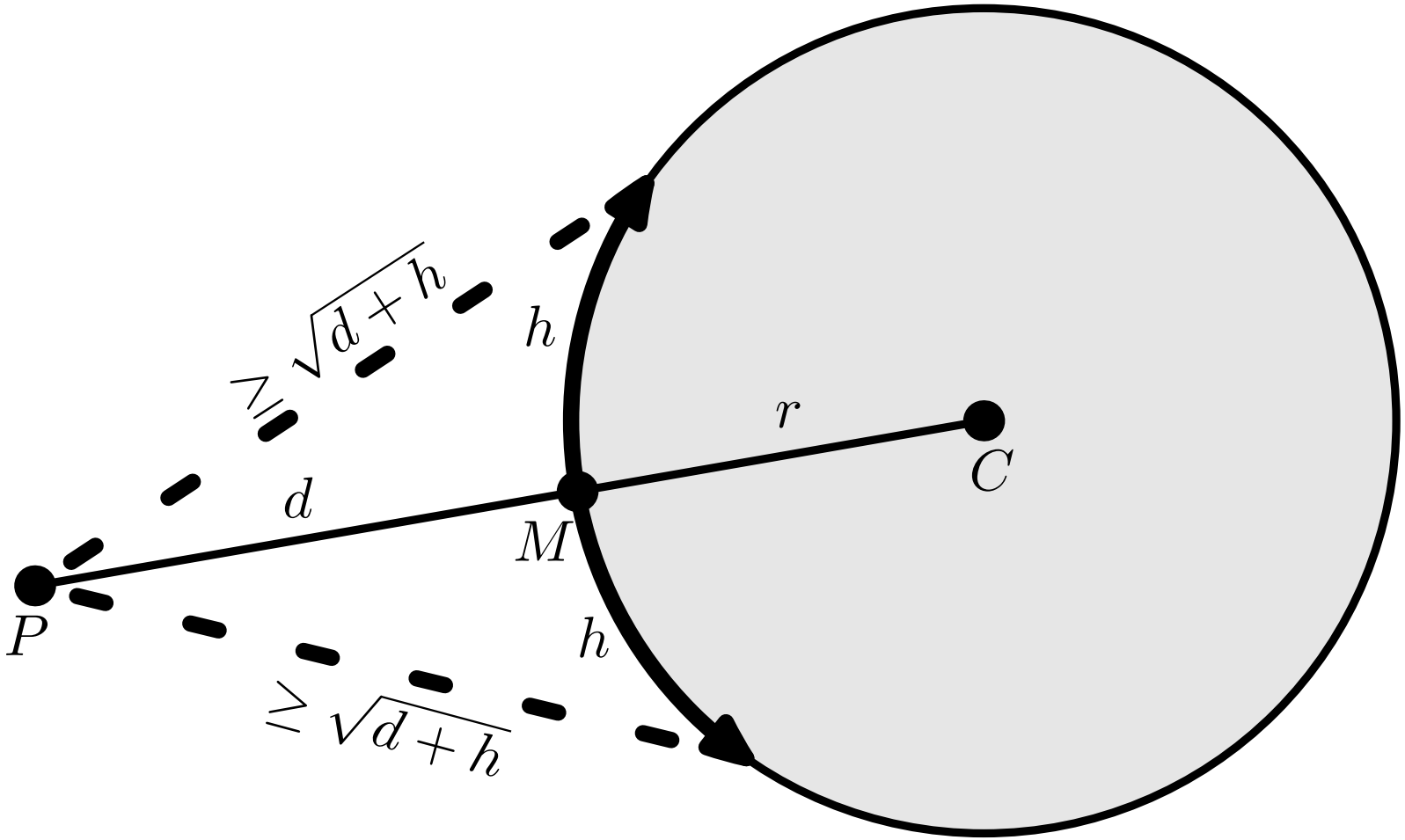
- Let us move along the circumference. How far can the desired integer point be?
- If we walk h units from M along the circumference, the distance from P will become more than $\sqrt{d^2 + h^2}$.
- When $h \geqslant 2\sqrt{d} + 1$, that distance is at least $d + 1$. So we are farther from P already than some integer point in the circle which is closest to M.

Illustration



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- ...tell us about your case!

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Problem D: Lost in Transfer

In this problem, you have to recover the numbers being transmitted, even in the unfortunate case one of them is lost.

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- You have to transmit each of them exactly once.
- You control the order of transmission.
- You then receive the same numbers in the same order, but one of them may be missing.
- Nevertheless, you have to recover the original set (order is irrelevant).

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- If they are equal, no element is missing.
- Otherwise, use the two statistics to determine the missing element.

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- Proceed until there are no more elements left.
- Example: to encode bits `10110...` (followed by zeroes) with the numbers 1, 2, ..., 20, print:

$$\underbrace{20, 19}_{for1}, \underbrace{1, 2}_{for0}, \underbrace{18, 17}_{for1}, \underbrace{16, 15}_{for1}, \underbrace{3, 4}_{for0}, \ldots, \underbrace{13, 14}_{for0}.$$

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- If no element is missing, inequalities come in pairs:
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- If an element is missing, some run of equal signs will have odd length.
- For example, if 17 is missing, we still have:
  $20 > 19 > 1 < 2 < 18 > 16 > 15 > 3 < 4 < \ldots < 13 < 14$.
- The way to decode it: take the next sign and transform $> \to 1$ and $< \to 0$; remove the sign from consideration; if the sign after it is the same, remove it too.

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- In the second run, look at the permutation q of the first 19 elements.

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- With high probability, there is only one possible y such that permuting according to $p(y)$ and then dropping one of the elements could have produced q.

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- Just check all $512 \cdot 20$ possibilities to be sure!

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- With high probability, there is only one possible y such that permuting according to $p(y)$ and then dropping one of the elements could have produced q.
- Just check all $512 \cdot 20$ possibilities to be sure!
- Then y is the value x you wanted to transfer.

Statement

Problem E: Maze with a Hint

In this problem, you have to first look at a maze and write a short hint, and then look at the hint and interactively solve the maze.

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  - Breadth-first search.
- How do we solve a maze when we only see the current square?
  - Right-hand rule: move along the maze so that you always keep touching the wall with your right hand.
  - Heuristic: maintain the map of visited squares, and move to the "best" unknown square, in some sense (closest to us, most probably on the right path, ...).

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- If we solve using the right-hand rule, the path will contain half the squares *on average*.

Packing the Optimal Path

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  Hint length: 1530, still too long.

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- Improved: Use one bit when there are 2 possible paths, two bits when there are 3.
  Hint length: 910, OK.

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- On each step, we have 1, 2, or 3 possible paths forward.
- If there is a single path forward, no hint is needed.
- Baseline: Remember each remaining step using two bits.
  Hint length: 1530, still too long.
- Improved: Use one bit when there are 2 possible paths, two bits when there are 3.
  Hint length: 910, OK.
- Big-integer: Use one bit when there are 2 possible paths, one ternary digit when there are 3. Pack all that into a big integer, and print it in binary as a hint.
  Hint length: 850, OK.

Remembering Critical Crossroads

Solution 2: walk using the right-hand rule, but take a hint at 4-crossroads.

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- Right-hand rule: hint length 430, path length 5530.

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- A 4-crossroads is a square with all 4 paths open.
- There will be only few 4-crossroads along our path.
- On each of them, remember the right direction (found by breadth-first search) using two bits.
- Right-hand rule: hint length 430, path length 5530.
- Left-hand rule: hint length 406, path length 4858.

Other Solutions Are Possible

Solution 3: (insert your solution here).

Statement

Problem F: Maharajas Are Going Home

In this problem, you have to find a winning move for a disjunctive sum of games, or determine that there is none.

Details

In this problem, you have to find a winning move for a disjunctive sum of games, or determine that there is none.

A maharaja has all the moves available to chess queen and chess knight. In this problem, it can move only towards the origin, and multiple maharajas can occupy the same square.

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A maharaja has all the moves available to chess queen and chess knight. In this problem, it can move only towards the origin, and multiple maharajas can occupy the same square.

You are given the initial positions of maharajas on a board which is infinite in positive directions.

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A maharaja has all the moves available to chess queen and chess knight. In this problem, it can move only towards the origin, and multiple maharajas can occupy the same square.

You are given the initial positions of maharajas on a board which is infinite in positive directions.

Two players move in turns, the player who can not make a move loses.

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A maharaja has all the moves available to chess queen and chess knight. In this problem, it can move only towards the origin, and multiple maharajas can occupy the same square.

You are given the initial positions of maharajas on a board which is infinite in positive directions.

Two players move in turns, the player who can not make a move loses.

Find the lexicographically smallest winning move in the following form: number of maharaja, destination row, destination column.

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A maharaja has all the moves available to chess queen and chess knight. In this problem, it can move only towards the origin, and multiple maharajas can occupy the same square.

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Two players move in turns, the player who can not make a move loses.

Find the lexicographically smallest winning move in the following form: number of maharaja, destination row, destination column.

Or determine there is no such move.

Grundy Numbers

The problem boils down to finding Grundy value for every possible position of a maharaja. Then, compute the xor-sum of the given maharajas. If it is zero, there is no winning move. Otherwise, iterate over all possible moves and find the lexicographically first move that makes the xor-sum zero.

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Straightforward solution, by definition: for every square, construct a set of Grundy values after all possible moves, and find its minimal excludant.

Bitsets

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- For each square, the slowest operation is to find the union of the respective three sets, then find its minimal excludant.

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- Maintain the sets for each row, column, and diagonal.
  When we learn the value in a new square, update these sets.
- For each square, the slowest operation is to find the union of the respective three sets, then find its minimal excludant.
- Speedup: store sets as bitsets.
  Then we effectively operate on 32 or 64 elements at once.

Optimizations

What if it is still too slow?

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  The table of Grundy values is also symmetric.

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Optimizations

What if it is still too slow?

- Symmetry: rows have the same sets as columns, lower diagonals have the same sets as upper diagonals.
  The table of Grundy values is also symmetric.
- When looking for minimal excludant in a row, start with the previous value for that row, instead of starting from 0 every time.
- When looking for the best move, consider only possible moves, instead of iterating over the whole board.

Statement

Problem G: Ook

In this problem, you have to cut pieces from a string, match them to a pattern, and maximize the sum of results.

Details

In this problem, you have to cut pieces from a string, match them to a pattern, and maximize the sum of results.

Example: $o = 3$, $k = 5$, $S = \texttt{ookkoko}$, $P = \texttt{ok?}$.

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Example: $o = 3$, $k = 5$, $S =$ `ookkoko`, $P =$ `ok?`.

`ookkoko`

`ok?` miss 1, value $3 + 3 + 5 = 11$, result $11/2^1 = 5$.

`ok?` miss 0, value $3 + 5 + 5 = 13$, result $13/2^0 = 13$.

`ok?` miss 1, value $5 + 5 + 3 = 13$, result $13/2^1 = 6$.

`ok?` miss 2, value $5 + 3 + 5 = 13$, result $13/2^2 = 3$.

`ok?` miss 0, value $3 + 5 + 3 = 11$, result $11/2^0 = 11$.

Details

In this problem, you have to cut pieces from a string, match them to a pattern, and maximize the sum of results.

Example: $o = 3$, $k = 5$, $S =$ `ookkoko`, $P =$ `ok?`.

`ookkoko`

`ok?` miss 1, value $3 + 3 + 5 = 11$, result $11/2^1 = 5$.

`ok?` miss 0, value $3 + 5 + 5 = 13$, result $13/2^0 = 13$.

`ok?` miss 1, value $5 + 5 + 3 = 13$, result $13/2^1 = 6$.

`ok?` miss 2, value $5 + 3 + 5 = 13$, result $13/2^2 = 3$.

`ok?` miss 0, value $3 + 5 + 3 = 11$, result $11/2^0 = 11$.

The best solution is to cut like this: o $\underbrace{\texttt{okk}}$ $\underbrace{\texttt{oko}}$.

This gives a total score of $13 + 11 = 24$.

Convolution

For each starting position i for the pattern in string S, the number of mismatches is the sum of two values, $A_i + B_i$:

- A_i is the number of `k` in the pattern when there is `o` in the string.
- B_i is the number of `o` in the pattern when there is `k` in the string.

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Let us calculate all numbers A_i (and then B_i similarly).

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- B_i is the number of `o` in the pattern when there is `k` in the string.

Let us calculate all numbers A_i (and then B_i similarly).

Consider a binary sequence "there is `o` in this position of S" and a binary sequence "there is `k` in this position of P".

Find their convolution using Fast Fourier Transform: reverse the latter, FFT both, multiply, inverse FFT the result.

The resulting values are the numbers A_i.

Dynamic Programming

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Dynamic Programming

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Then $f(i) = \max\{f(i-1), f(i-|P|) + v(i-|P|)\}$.
Here, $v(j)$ is the value obtained by matching the pattern starting at position j of the string.

Statement

Problem H: Pi Approximation

In this problem, you have to find the best approximation for π obtained by calculating the number of Pythagorean triples from integers $1, 2, \ldots, n$.

General Outline

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For example, there's Euclid's formula: for $0 < n < m$ where n and m are coprime and of different parity, $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$ are a unique primitive Pythagorean triple.

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It turns out that the number of such triples with each possible sum is small enough (as hinted by the example), so we can store these quantities in a straightforward way, in a 200MB array of bytes.

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It turns out that the number of such triples with each possible sum is small enough (as hinted by the example), so we can store these quantities in a straightforward way, in a 200MB array of bytes.

To speed things up, process all queries at once.

Statement

Problem I: Partition of Queries

In this problem, you have to decide when to rebuild a data structure so that the total time for processing the given queries is minimized.

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In this problem, you have to decide when to rebuild a data structure so that the total time for processing the given queries is minimized.

Example: queries are "++??+?", rebuild cost is $y = 5$.

- Without interfering, we get the total cost $2 + 2 + 3 = 7$.
- We put a rebuild after two adds: "++|??+?", then the cost is $5 + 0 + 0 + 1 = 6$.

Dynamic Programming 1

Compute the prefix sums:
$p(i)$ is the number of "+" up to position i,
$r(i)$ is the number of "?" up to position i.
Positions are numbered from 1 to n, prefix sums from 0 to n.

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Let $c(i)$ be the cost to process the first i queries without rebuilds:
The $c(i) = c(i-1)$, plus $p(i)$ if the i-th query is a "?".

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The $c(i) = c(i-1)$, plus $p(i)$ if the i-th query is a "?".

Let $s(j, i)$ be the cost to process all queries from the segment from $j+1$ to i inclusive without rebuilds.
Then $s(j, i) = c(i) - c(j) - p(i) \cdot (r(j) - r(i))$. Indeed, we consider only the $(r(j) - r(i))$ "?" on the respective positions, and for each of them, we have to subtract $p(i)$ "+" before that position.

Dynamic Programming 2

Finally, let $f(i)$ be the minimum possible cost to process the first i queries:

- we either process all of them, or
- insert a rebuild after position $j < i$, take $f(j)$ into account, and process all queries from the segment from $j + 1$ to i inclusive without rebuilds.

Dynamic Programming 2

Finally, let $f(i)$ be the minimum possible cost to process the first i queries:

- we either process all of them, or
- insert a rebuild after position $j < i$, take $f(j)$ into account, and process all queries from the segment from $j + 1$ to i inclusive without rebuilds.

So, $f(i) = \min\{c(i), c(j) + s(j, i) + y \text{ for all } j < i\}$.

The answer we want is $f(n)$.

Dynamic Programming 3

Let us make some adjustments to the formula:
$f(i) = \min\{c(i), b(j,i) \text{ for all } j < i\}$, where $b(j,i) = c(j) + s(j,i) + y$.
Naturally,
$b(j,i) = c(j) + c(i) - c(j) - p(i) \cdot (r(j) - r(i)) = c(i) + p(i) \cdot r(i) - p(i) \cdot r(j)$.

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Now consider everything depending only on i as a separate function:
$g(i) = c(i) + p(i) \cdot r(i)$.
Then $g(i) - b(j,i)$ is just $p(i) \cdot r(j)$.

Note that $p(i)$ and $r(j)$ are nondecreasing sequences.
So it turns out we can use some "convex hull" optimization to compute the minimums efficiently: do a binary search for the respective j, or do two pointers, as when i increases, j can not decrease.

Statement

Problem J: Random Chess Game

In this problem, you have to win 150 *chess games against an opponent who picks random moves, and all possible moves are listed for you each time you have to make a move.*

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- There are two sources for heuristics:
  - We know chess rules.
  - We know the opponent picks moves uniformly at random.
- Be ready to iterate, and likely not have your solution accepted on the first try.

Heuristics

- If there is a checkmate move, do it!

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- Prefer moving pawns to moving heavier pieces...

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- We strive to take Black's pieces... but we don't like to put the Black King in check: too little responses, too high the probability of a good response.
- Prefer moving pawns to moving heavier pieces... but avoid moving pawns adjacent to the White King, just to keep the King safe.

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- Strive for promoting your pawns.

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- Avoid placing heavy pieces adjacent to the Black King.

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- Avoid repeated positions, which can be indirectly tracked as their sets of possible moves.

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- In the endgame, minimize distance between Kings to avoid draws by 50-move rule...

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- Strive for promoting your pawns.
- Avoid placing heavy pieces adjacent to the Black King.
- Avoid repeated positions, which can be indirectly tracked as their sets of possible moves.
- In the endgame, minimize distance between Kings to avoid draws by 50-move rule... but be careful not to produce a stalemate.

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- For example: `1.e4, 2.Nf3, 3.Bc4, 4.Ne5, 5.Qf3`.
- If this does not work, consider moving the Queen back, then proceed normally with heuristics.

Statement

Problem K: What? Subtasks? Again?

In this problem, you have to pick a subset of contest features so that the number of satisfied contestants is the maximum possible integer $\leqslant m$.

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For each of the $2^5 = 32$ possible sets of features, count the contestants satisfied by this set.

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Pick the greatest number of contestants $\leqslant m$.

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For each contestant, compute the mask of features they don't like.

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Pick the greatest number of contestants $\leqslant m$.

If there is none, print the required line instead of the number.

Statement

Problem L: The Five Bishops

In this problem, you have to checkmate or stalemate a lone black King with five white bishops on an infinite chessboard.

Details

In this problem, you have to checkmate or stalemate a lone black King with five white bishops on an infinite chessboard.

- If we don't have two bishops of each color (two on light squares and two on dark squares), we can't do it.

Details

In this problem, you have to checkmate or stalemate a lone black King with five white bishops on an infinite chessboard.

- If we don't have two bishops of each color (two on light squares and two on dark squares), we can't do it.
- Four bishops are sufficient.

Getting Far

Step 1: Get far from the King.

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- The King can't attack both colors simultaneously.
- Defend one bishop of the attacked color with another.
- We can lose the third bishop of some color, or just move it away.
- Bearing that in mind, move the bishops away (say, 10^6 squares) from the King.

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Step 2: Bound the King to one diagonal line.

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- Enumerate them by integers, for example, $c = x - y$.
- Let the King be on diagonal k.
- Then place the bishops on diagonals $k - 10$, $k - 9$, $k + 9$, and $k + 10$.
- As a result, the King can't get to diagonals $k + x$ where $|x| \geqslant 9$.

Controlling the Diagonals

Step 2: Bound the King to one diagonal line.

- Consider one of the two types diagonals of the board.
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- As long as there is more than one diagonal left for the King, pick the bishop on the diagonal farthest from the King, and move it one step closer.

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- As a result, the King can't get to diagonals $k + x$ where $|x| \geqslant 9$.
- As long as there is more than one diagonal left for the King, pick the bishop on the diagonal farthest from the King, and move it one step closer.
- The King won't be attacked by this move, but the number of diagonals he can visit will decrease by one.

Stalemate

Step 3: Produce a stalemate.

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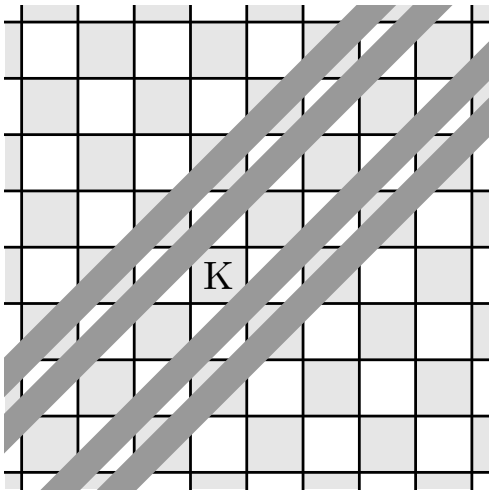

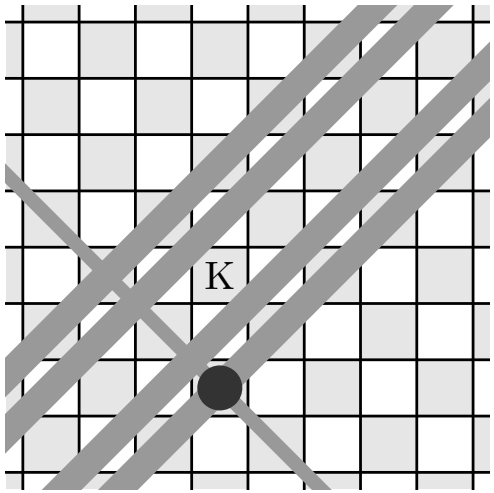

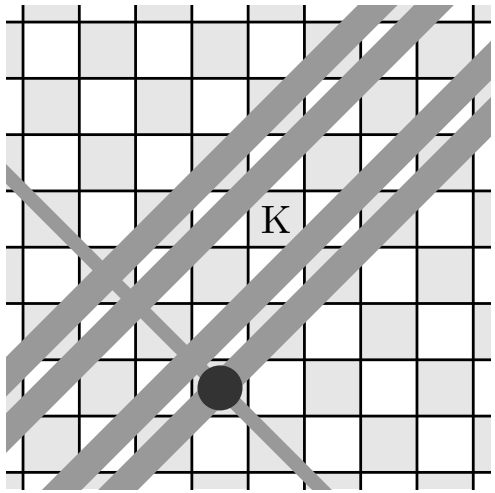

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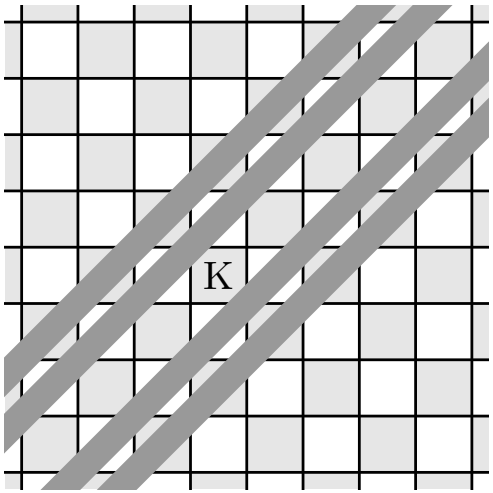

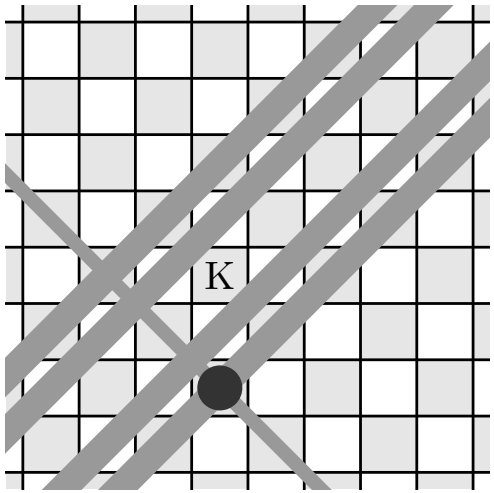

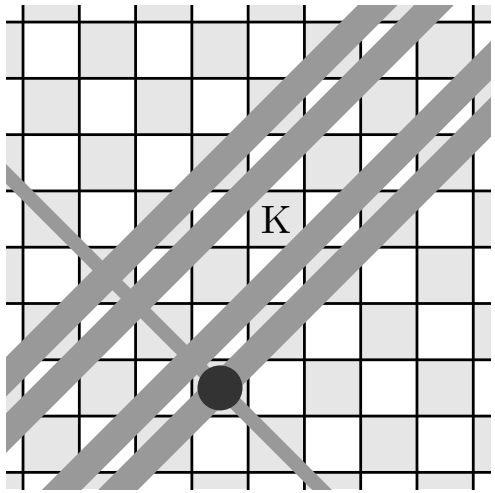

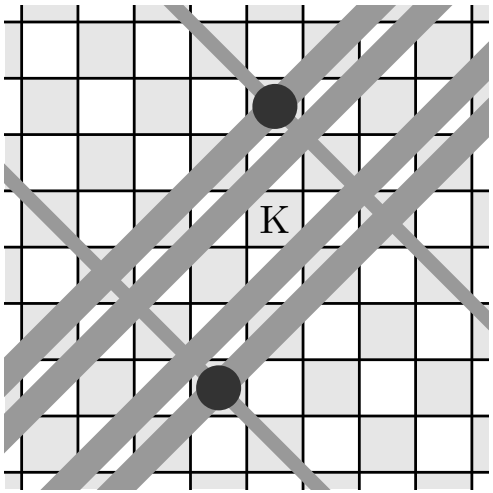

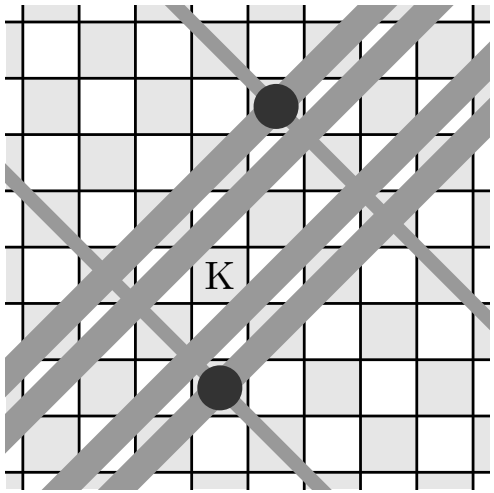

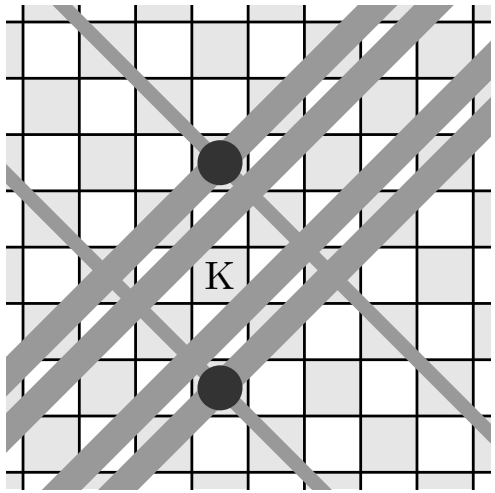

Based on LV St. Petersburg State University Championship

Contest Developers:

- Ivan Kazmenko
- Aleksandr Logunov
- Andrei Lopatin
- Anton Maydell
- Artur Riazanov

Questions?